Equilibrium liquidity in closing auctions, opening auctions, and continuous trading

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Abstract

In recent years, closing auctions have attracted substantial trading volume at low price impact, while opening auctions remain comparatively illiquid relative to the continuous trading period. We study this contrast in a continuous-time Kyle model with opening and closing auctions and $N \geq 1$ competing, identically informed insiders. We construct a symmetric linear equilibrium when closing-auction noise trading is sufficiently large relative to daytime noise trading, a condition that holds automatically with one insider. Imminent public disclosure of overnight news induces aggressive trading that can make the opening auction less liquid than the subsequent continuous trading period. In contrast, the closing auction sustains gradual information revelation during the day and lowers price impact to $2/(N+1)^2$ of its pre-auction level, while leaving the fundamental value uncertain. Calibration of the model to equity data illustrates how greater assumed insider competition can lead to higher initial price impact but lower impact near the close, matching empirical trends.

Keywords: Kyle model, Informed trading, Liquidity, Price Impact, Closing auction, Opening auction

1 Introduction

Opening and closing auctions allow for concentrated trading on either side of an otherwise continuous trading day, yet their levels of liquidity differ substantially. Goyal, Jegadeesh, and Wu [2025] find that closing auctions have approximately three times lower price impact than the continuous trading period, whereas opening auctions are comparatively illiquid. This relates to the increased volume seen in closing auctions in recent years. Bogousslavsky and Muravyev [2023] document an increase in the closing-auction share of US daily trading volume from 3.1% in 2010 to 7.5% in 2018. More recent exchange evidence underscores the growing importance of these auctions, with Bazinas [2024] reporting that US closing auctions accounted for 9.44% of total equity notional traded in the second quarter of 2024, matching an average of $55.5 billion per day.[1] For Europe, Besson and

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[1] In contrast, opening auctions have less volume and have decreased in popularity, making up just over 2% of US trading volume in 2012/2013 and under 1.5% by 2021; c.f. Goyal, Jegadeesh, and Wu [2025].

Quily [2026] report that closing auctions account for more than 25% of multilateral addressable volume in early 2026. These market trends highlight the importance of understanding how auctions interact with the rest of the trading day. In this paper we develop an equilibrium model in the the spirit of Kyle [1985] that endogenizes the liquidity in the opening and closing auctions as well as the intermediate continuous trading phase. This model helps (i) explain how an opening auction can be relatively costly, while a closing auction absorbs substantial order flow at lower impact than the continuous trading period and (ii) understand how the opportunity to trade at the close affects informed trading and price discovery throughout the trading day.

We address these questions by developing an equilibrium model that combines an opening auction, continuous trading, and a closing auction with strategically competing insiders and market makers. Building on the seminal papers of Kyle [1985] and Back [1992], we consider N risk-neutral insiders who share knowledge of an asset's fundamental value and trade strategically against noise traders. Perfectly competitive market makers observe aggregate order flow and set prices equal to conditional expectations of the asset's fundamental value given their information. Noise trading evolves continuously during the continuous trading period, while involving discrete orders at the opening and closing auctions. [2] A portion of the insiders' information becomes public immediately after the opening auction, reflecting the dissemination of overnight news, while their remaining informational edge stays private throughout the trading day. This distinction allows the model to capture the different environments under which opening and closing auctions operate, and turns out to be an important driver of opening auction illiquidity in our model.

Our main result constructs a symmetric linear equilibrium in which informed trading, price impact, and the market makers' residual uncertainty are jointly determined across all three trading periods. In the general case, the equilibrium is explicit up to the solution of a scalar equation governing information revelation at the opening auction, while the version of the model that omits the opening auction leads to entirely explicit formulas. With one insider, the construction always applies, while with multiple insiders, it requires sufficiently large closing-auction noise-trading variance relative to continuous-trading noise variance, with a threshold that depends on the number of insiders. The closing auction provides a final opportunity for insiders to exploit their information and thereby allows for gradual trading in the case of multiple identically informed insiders during the continuous trading period. This is in strike contrast to the case without a closing auction, where competition among multiple insiders leads to private information being revealed immediately Holden and Subrahmanyam [1992].

At the close, equilibrium price impact falls to $2/(N+1)^2$ times its level immediately before the auction. With one insider, impact is constant during continuous trading and halves at the close, while with multiple insiders it declines during the continuous trading period and experiences a further drop during the closing auction.[3] The closing auction also reveals a fraction $N/(N+1)$ of the uncertainty remaining immediately beforehand, but leaves strictly positive residual uncertainty; that is, the fundamental price is not fully revealed.

The opening auction illustrates how the timing of public disclosure shapes liquidity. Insiders trade on both components of their information, but place greater weight on the component that becomes public immediately afterwards. They must exploit this information at the open, whereas they can continue to profit from the persistent component later in the day. Consequently, in

[2] Including noise trading in auctions is consistent with the evidence of Jegadeesh and Wu [2022], who document a positive association between closing-auction volume and proxies for uninformed trading, including ETF arbitrage activity and flows into and out of index mutual funds.

[3] The latter is consistent with intraday price impact trends; c.f. [Cont, Kukanov, and Stoikov, 2014, Figure 5].

equilibrium, the opening auction can have higher price impact than the start of continuous trading when the variance of information about to become public is sufficiently large relative to persistent uncertainty, or when opening noise trading is sufficiently small relative to subsequent noise trading. Without the dissemination of overnight news this ordering does not arise in our equilibrium.

Our single-insider equilibrium also shows that reallocating noise-trading variance from the continuous trading period to the close, while holding its sum and all other model parameters fixed, leaves liquidity during continuous trading unchanged. Indeed, the constant continuous-market price impact coefficient depends on the daytime and closing noise-trading variances only through their sum. However, a reallocation of noise trading activity does change the timing of information revelation and the uncertainty remaining at the close. This observation suggests that a greater concentration of noise trading at the close need not make daytime trading more costly. It is consistent with the absence of a universal deterioration in continuous-market liquidity found in the empirical literature. Concretely, Aitken, Comerton-Forde, and Frino [2005] find that introducing a closing auction reduces late continuous-trading volume without materially changing continuous-market spreads. Kandel, Rindi, and Bosetti [2012] find that its effects are concentrated near the close, including reductions in spreads and volatility, while Comerton-Forde and Rindi [2022] find no evidence that increased auction activity harms continuous-trading market quality.

The present paper also contributes to the literature on strategic informed trading by connecting discrete auctions and continuous trading through a common information and pricing problem. The study of competition among informed traders in continuous time dates back to with further studies allows insiders to have different information flows and risk aversions pioneered by Foster and Viswanathan [1996] and Back, Cao, and Willard [2000] respectively. The auctions present in our model help overcome the front-running behaviour of the strategically competing insiders with perfect information, which precluded equilibrium construction in the purely continuous trading models.[4]

Regarding how impact scales with order size, Salek, Challet, and Muni Toke [2023] document distinct regimes of auction price impact, including a substantial linear region, and their subsequent work [Salek, Challet, and Muni Toke, 2024] models the accumulation of auction liquidity through order-book dynamics. We instead determine an endogenous level of linear impact from strategic informed trading and competitive filtering, and link the resulting auction outcomes to the entire trading day.

The paper is organized as follows. Section 2 introduces the model. Section 3 presents our main results, including the symmetric equilibrium and its principal special cases. Section 4 then discusses liquidity and information revelation across the trading day, while Section 5 performs a calibration of the model, with the opening auction omitted, on equity data. The derivation of the equilibrium and proofs of the main results are deferred to Appendix A.

2 Problem setup

Here we consider a version of Kyle's model that has continuous-time trading at times $t \in (0, 1)$ together with an opening auction at time $t = 0$ and a closing auction at time $t = 1$. In our model, a portion of the information will become publicly available immediately after the opening auction ends to reflect dissemination of overnight news. As such, we initiate the model at time $0-$ prior to

[4]This contrasts with discrete trading models, where such equilibria are possible with perfect competition; see e.g. Subrahmanyam [1998].

the opening auction at time 0 and write $\mathbb{T} = \{0-\} \cup [0,1]$. We assume there is a single risky asset with price process $(P_t; t \in \mathbb{T})$ whose fundamental value V satisfies

$$V = V_o + V_c, \qquad \text{where} \quad V_o \sim N(\mu_o, \sigma^2_{V,o}) \text{ and } V_c \sim N(\mu_c, \sigma^2_{V,c})$$

for some $\mu_o, \mu_c \in \mathbb{R}$ and $\sigma_{V,o}, \sigma_{V,c} > 0$, and where V_o is independent of V_c. We refer to V_o as the *overnight component* and V_c as the *persistent component*, which is only revealed after the closing auction a time T.

The key difference from the standard continuous-time Kyle model of Back [1992] is the presence of auctions, where noise traders place bulk trades. Concretely, we assume their position in the stock is given by

$$Z_t = \Delta Z_0 + \sigma_{Z,m} B_t + \Delta Z_1 \mathbf{1}_{\{t=1\}}, \qquad Z_{0-} = 0,$$

where $\Delta Z_0 \sim N(0, \sigma^2_{Z,o})$, $(B_t;\, t \in [0,1])$ is a standard Brownian motion independent from V and ΔZ_0, and $\Delta Z_1 \sim N(0, \sigma^2_{Z,c})$ is independent from ΔZ_0, B and V, and $\sigma_{Z,o}, \sigma_{Z,m}, \sigma_{Z,c} > 0$ are given constants. Throughout, the subscripts o, m and c denote the opening auction, main (i.e., continuous) trading period and closing auction respectively. We see that noise traders place a bulk trade ΔZ_0 in the opening auction, then trade continuously during the day according to a Brownian motion, and place a final bulk trade ΔZ_1 at the closing auction.

Next, we assume there are $N \in \mathbb{N}$ insiders. Each insider has the same information, knowing both V_o and V_c at time $0-$, and observes the evolution of the price process P. Hence, their common filtration $(\mathcal{F}^I_t)_{t \in \mathbb{T}}$ is given by

$$\mathcal{F}^I_t = \sigma(V_o) \vee \sigma(V_c) \vee \sigma(P_s; 0 \le s < t), \qquad t \in \mathbb{T}.$$

Here, $\mathcal{F}^I_t$ denotes the insider's information available when submitting an order at time t. In particular, auction orders cannot depend on the corresponding clearing prices. Insider i's $\mathcal{F}^I$-adapted holdings process is denoted by $(X^i_t;\, t \in \mathbb{T})$ for $i = 1, \dots, N$ with initial value $X^i_{0-} = 0$. As such, the values P_0 and X^i_0 represent the asset's price and insider i's holdings respectively upon completion of the opening auction. We restrict attention to absolutely continuous strategies during the continuous trading period and allow block orders only in the opening and closing auctions. In other words, we assume that each process X^i is absolutely continuous on $(0,1)$, but can experience a jump at the times $t = 0$ and $t = 1$, when the insider participates in the opening and closing auctions, respectively. This leads to the representation

$$X^i_t = \Delta X^i_0 + \int_0^t \theta^i_s ds + \Delta X^i_1 \mathbf{1}_{\{t=1\}}, \quad t \in [0,1], \qquad i = 1, \dots, N,$$

for the opening auction bulk trade ΔX^i_0, the continuous period trading rate $(\theta^i_s;\, 0 < s < 1)$ and closing auction bulk trade ΔX^i_1. When the problem is initiated at time $t \in \mathbb{T}$, we denote by $\mathcal{A}_t$ the set of all such processes $(X^i_s;\, t \le s \le 1)$ which are square integrable. We assume all of the insiders are risk-neutral, meaning their common value function is

$$v(t, p, V_o, V) = \sup_{X^i \in \mathcal{A}_t} J(t, p, V_o, V, X^i), \qquad t \in \mathbb{T}, \quad p, V_o, V \in \mathbb{R},$$

where

$$J(t,p,V_o,V,X^i) = \mathbb{E}\bigg[\int_{(t,1]}(V-P_s)dX^i_s \,\bigg|\, \mathcal{F}^I_t\bigg]$$
$$= \mathbb{E}\bigg[(V-P_0)\Delta X^i_0 \mathbf{1}_{\{t=0-\}} + \int_{t\vee 0}^1 (V-P_s)\theta^i_s ds + (V-P_1)\Delta X^i_1 \mathbf{1}_{\{t<1\}} \,\bigg|\, \mathcal{F}^I_t\bigg] \tag{2.1}$$

and $P_t = p$. Here we tacitly omitted the dependence on the other insiders' strategies X^{-i} because in the sequel we will only consider the symmetric case $X^i = X^j$ for all i, j. We are interested in this stochastic control problem at time $t = 0-$ with initial uninformed price $\mu = \mu_o + \mu_c$.

Next, we assume the market maker is perfectly competitive. That is, they set the price to be a martingale under their own filtration $\mathcal{F}^M$,

$$P_t = \mathbb{E}[V \mid \mathcal{F}^M_t], \qquad t \in \mathbb{T}.$$

The market maker cannot differentiate between the orders of the insiders and the noise traders, so their filtration is generated by the total order flow

$$Y_t = \sum_{i=1}^N X^i_t + Z_t$$

together with the publicly revealed information after the opening auction; i.e.,

$$\mathcal{F}^M_{0-} = \{\emptyset, \Omega\}, \quad \mathcal{F}^M_0 = \sigma(\Delta Y_0), \quad \mathcal{F}^M_{0+} = \sigma(\Delta Y_0, V_o), \quad \mathcal{F}^M_t = \sigma(V_o) \vee \sigma(Y_s;\, 0 \le s \le t), \quad t \in (0,1].$$

This information structure will lead to the price process P being neither left- nor right-continuous at $t = 0$. Indeed, the market maker will set prices in the opening auction at time $t = 0$ based on the observed order flow ΔY_0 and then upon conclusion of the auction will again update the price by incorporating the revealed value of V_o.

As is typical, we will look for solutions in the class of linear pricing rules:

$$P_t = P_{0-} + \lambda_o \Delta Y_0 \mathbf{1}_{\{t\ge 0\}} + \big(\kappa_o(V_o - \mu_o) + \kappa_r \Delta Y_0\big)\mathbf{1}_{\{t>0\}} + \int_{(0,t)} \lambda_s dY_s + \lambda_c \Delta Y_1 \mathbf{1}_{\{t=1\}}$$
$$= \mu_o + \mu_c + \lambda_o\bigg(\sum_{i=1}^N \Delta X^i_0 + \Delta Z_0\bigg)\mathbf{1}_{\{t\ge 0\}} + \kappa_o(V_o - \mu_o)\mathbf{1}_{\{t>0\}} + \kappa_r\bigg(\sum_{i=1}^N \Delta X^i_0 + \Delta Z_0\bigg)\mathbf{1}_{\{t>0\}}$$
$$+ \int_0^t \lambda_s \sum_{i=1}^N \theta^i_s ds + \int_0^t \sigma_{Z,m}\lambda_s dB_s + \lambda_c\bigg(\sum_{i=1}^N \Delta X^i_1 + \Delta Z_1\bigg)\mathbf{1}_{\{t=1\}} \tag{2.2}$$

for some deterministic positive process $(\lambda_s;\, s \in (0,1))$, as well as constants $\lambda_o, \lambda_c > 0$ and $\kappa_o, \kappa_r \in \mathbb{R}$. Here λ_o is the impact generated from the opening auction. Then, once V_o is publicly revealed, the market maker reacts and updates the price. Since V_o and ΔY_0 are dependent, we need to incorporate two constants κ_o and κ_r modulating the influence on the price jump at $t = 0+$ from the information revelation and from the previously observed order flow, respectively. The process $(\lambda_s;\, s \in (0,1))$ is the standard price impact process in the continuous trading period and λ_c is the price impact constant for the closing auction.

3 Main results

In this market we look for a symmetric equilibrium, which is defined as follows.

Definition 3.1 (Symmetric equilibrium)**.** *A symmetric equilibrium consists of*

(i) an admissible strategy $X \in \mathcal{A}_{0-}$ used by all the insiders,
(ii) nonnegative price impact coefficients $\lambda_o, (\lambda_t)_{t\in(0,1)}$ and λ_c, and
(iii) information revelation pricing coefficients $\kappa_o, \kappa_r \in \mathbb{R}$,

such that

1. *(Insider's optimality) The strategy X is optimal for each insider when all of the other insiders also use X. That is, $v(0-, \mu, V_o, V) = J(0-, \mu, V_o, V, X)$,*
2. *(Perfect competition) The price process P of (2.2) satisfies $P_t = \mathbb{E}[V \mid \mathcal{F}_t^M]$ for every $t \in \mathbb{T}$.*

Here, symmetry refers to the fact that all of the insiders use the same strategy. In the present context, where they all have the same information flow $\mathcal{F}^I$, this is a natural class in which to search for an equilibrium. The following result constructs a symmetric equilibrium in this setting when there is sufficient noise trading volume in the closing auction, relative to the number of insiders and noise trading volume in the continuous trading period.

Theorem 3.2 (Main result)**.** *Suppose that the parameter condition*

$$\frac{\sigma_{Z,c}^2}{\sigma_{Z,m}^2} > \frac{(N+1)^2(3N-4)}{4} \tag{3.1}$$

holds. Define

$$u = 1 - \frac{(N+1)^2(3N-4)\sigma_{Z,m}^2}{4\sigma_{Z,c}^2} > 0, \qquad \sigma_{Z,R} = \frac{2\sigma_{Z,c}\sqrt{u}}{\sqrt{N}(N+1)\sigma_{Z,o}}, \tag{3.2}$$

and let $x^* \in (0,1)$ be the unique solution of the equation

$$\sigma_{V,c}\Big(2Nx\big(1-\sigma_{Z,R}\sqrt{1-x}\big)+(N-1)(1-x)\Big) = \sqrt{(1-x)\big((N+1)^2\sigma_{V,c}^2(1-x)+4N\sigma_{V,o}^2\big)}. \tag{3.3}$$

Then the model admits a symmetric equilibrium given by the following specifications:

- *The price impact coefficients in the opening auction, continuous trading period and closing auction are*

$$\begin{aligned}
\lambda_o &= \frac{N\sigma_{V,c}\sqrt{x^*}(1-\sigma_{Z,R}x^*\sqrt{1-x^*})}{(N+1)\sigma_{Z,o}\sqrt{1-x^*}},\\
\lambda_t &= \frac{(N+1)\sqrt{N}\sigma_{V,c}\sqrt{x^*}u^{N/(6N-8)}}{2\sigma_{Z,c}(u+(1-u)t)^{2(N-1)/(3N-4)}}, \qquad t\in(0,1),\\
\lambda_c &= \frac{2}{(N+1)^2}\lambda_{1-} = \frac{\sqrt{N}\sigma_{V,c}\sqrt{x^*}u^{N/(6N-8)}}{(N+1)\sigma_{Z,c}}.
\end{aligned} \tag{3.4}$$

- *The information-revelation pricing coefficients are*

$$\kappa_o = \frac{x^*(1-\sigma_{Z,R}\sqrt{1-x^*})}{1-\sigma_{Z,R}x^*\sqrt{1-x^*}},$$
$$\kappa_r = \frac{\sigma_{V,c}\sqrt{x^*}(1-(N+1)x^*+N\sigma_{Z,R}x^*\sqrt{1-x^*})}{(N+1)\sigma_{Z,o}\sqrt{1-x^*}}.$$

- *Each insider $i = 1, \dots, N$ follows*

$$\Delta X_0 = \alpha_o(V_o - \mu_o) + \alpha_c(V_c - \mu_c), \tag{3.5}$$
$$\theta_t = \frac{(N+1)\sigma_{Z,m}^2}{2\sqrt{N}\sigma_{V,c}\sigma_{Z,c}\sqrt{x^*}u^{N/(6N-8)}(u+(1-u)t)^{(N-2)/(3N-4)}}(V-P_t), \quad t \in (0,1), \tag{3.6}$$
$$\Delta X_1 = \frac{\sigma_{Z,c}}{\sqrt{N}\sigma_{V,c}\sqrt{x^*}u^{N/(6N-8)}}(V-P_{1-}),$$

where

$$\alpha_o = \frac{\sigma_{Z,o}\sqrt{1-x^*}}{N\sigma_{V,c}\sqrt{x^*}(1-\sigma_{Z,R}x^*\sqrt{1-x^*})}, \qquad \alpha_c = \frac{\sigma_{Z,o}\sqrt{1-x^*}}{N\sigma_{V,c}\sqrt{x^*}}. \tag{3.7}$$

The constants x^* and u determine the market maker's residual variance $\Sigma_t = \mathrm{Var}(V \mid \mathcal{F}_t^M)$ for the fundamental price V that is revealed from trading in the opening auction and continuous trading period. The next proposition records Σ in equilibrium.

Proposition 3.3 (Market maker's residual variance)**.** *Assume* (3.1) *holds. In the equilibrium of Theorem 3.2 we have* $\Sigma_{0-} = \sigma_{V,o}^2 + \sigma_{V,c}^2$, $\Sigma_0 = \frac{1}{N+1}(\sigma_{V,o}^2 + \sigma_{V,c}^2(1+N\sigma_{Z,R}x^*\sqrt{1-x^*}))$,

$$\Sigma_t = x^*\sigma_{V,c}^2\left(\frac{u}{u+(1-u)t}\right)^{\frac{N}{3N-4}}, \quad t \in (0,1), \qquad \textit{and} \qquad \Sigma_1 = \frac{x^*\sigma_{V,c}^2 u^{N/(3N-4)}}{N+1}.$$

In particular,

$$\Sigma_{0+} = x^*\sigma_{V,c}^2, \qquad \Sigma_{1-} = x^*\sigma_{V,c}^2 u^{N/(3N-4)}, \qquad \Sigma_1 = \frac{\Sigma_{1-}}{N+1}. \tag{3.8}$$

As such $1 - x^* \in (0,1)$ is the fraction of the persistent component's prior variance revealed by the opening auction. The fraction $u^{N/(3N-4)} \in (0,1)$ determines how much further this variance is reduced by the end of the continuous trading period. Because of the existence of the closing auction, we do not have $\Sigma_{1-} = 0$ as in the standard continuous-time Kyle model. Indeed, since the trading rate coefficient is bounded, which is the case because the denominator in (3.6) is bounded, we see that the insider does not trade according to a Brownian bridge here. Moreover, the true price V is not fully revealed even after the closing auction. This is because noise traders place a bulk trade in this final trading period. As in the standard one-period Kyle model, since noise traders make up a positive proportion of trading volume, the fundamental value V is obfuscated.

We now record a few special and important cases as corollaries. One such situation is when there is only one insider, in which case the condition (3.1) always holds. As such, the model admits an equilibrium irrespective of the relative noise trading volumes in the closing and continuous trading periods.

Corollary 3.4 (One insider)**.** *When there is one insider ($N = 1$), the parameter restriction* (3.1) *holds automatically. Define*

$$\sigma_{Z,R}^2 = \frac{\sigma_{Z,m}^2 + \sigma_{Z,c}^2}{\sigma_{Z,o}^2},$$

and let $x^* \in (0,1)$ be the unique solution of

$$\sigma_{V,c} x\big(1 - \sigma_{Z,R}\sqrt{1-x}\big) = \sqrt{(1-x)\big(\sigma_{V,c}^2(1-x) + \sigma_{V,o}^2\big)}.$$

Then the equilibrium of Theorem 3.2 reduces to the following specifications:

- *The price impact coefficients are*

$$\lambda_o = \frac{\sigma_{V,c}\sqrt{x^*}\big(1 - \sigma_{Z,R}x^*\sqrt{1-x^*}\big)}{2\sigma_{Z,o}\sqrt{1-x^*}}, \qquad \lambda_t = \lambda_m := \frac{\sigma_{V,c}\sqrt{x^*}}{\sqrt{\sigma_{Z,m}^2 + \sigma_{Z,c}^2}}, \qquad \lambda_c = \frac{\lambda_m}{2}.$$

  In particular, price impact is constant during the continuous trading period.

- *The information-revelation coefficients are*

$$\kappa_o = \frac{x^*\big(1 - \sigma_{Z,R}\sqrt{1-x^*}\big)}{1 - \sigma_{Z,R}x^*\sqrt{1-x^*}}, \qquad \kappa_r = \lambda_o(1 - 2\kappa_o).$$

- *The insider's equilibrium trades are*

$$\begin{aligned}
\Delta X_0^* &= \frac{V_o - \mu_o}{2\lambda_o} + \frac{\sigma_{Z,o}\sqrt{1-x^*}}{\sigma_{V,c}\sqrt{x^*}}(V_c - \mu_c),\\
\theta_t^* &= \frac{\sigma_{Z,m}^2}{\lambda_m(\sigma_{Z,c}^2 + \sigma_{Z,m}^2(1-t))}(V - P_t), \qquad t \in (0,1),\\
\Delta X_1^* &= \frac{V - P_{1-}}{2\lambda_c}.
\end{aligned}$$

- *The residual variance is $\Sigma_{0-} = \sigma_{V,o}^2 + \sigma_{V,c}^2$, $\Sigma_0 = \frac{1}{2}(\sigma_{V,o}^2 + \sigma_{V,c}^2(1 + \sigma_{Z,R}x^*\sqrt{1-x^*}))$*

$$\Sigma_t = \lambda_m^2\big(\sigma_{Z,c}^2 + \sigma_{Z,m}^2(1-t)\big), \quad t \in (0,1), \qquad \Sigma_1 = \frac{\Sigma_{1-}}{2} = \frac{\lambda_m^2\sigma_{Z,c}^2}{2}.$$

Next, we examine what happens when one omits the opening auction. In this case, the model effectively starts at time 0+ immediately after the public revelation of V_o, and the coefficients $\lambda_o, \kappa_o, \kappa_r$ do not need to be determined in equilibrium. Formally this choice corresponds to the limiting regime $\sigma_{Z,o} \downarrow 0$, leading to $x^* \uparrow 1$. Since no opening order flow conveys information about V_c, the initial conditional variance is $\Sigma_{0+} = \sigma_{V,c}^2$. Consequently, in the case when the parameter condition (3.1) holds, the equilibrium is entirely explicit.

Corollary 3.5 (No opening auction)**.** *Suppose the opening auction is omitted and V_o is publicly revealed before continuous trading begins, so that $P_{0+} = V_o + \mu_c$ and $\Sigma_{0+} = \sigma_{V,c}^2$. Assume* (3.1) *and*

define u as in Theorem 3.2. The market admits a symmetric linear equilibrium with price impact coefficients

$$\lambda_t = \frac{(N+1)\sqrt{N}\,\sigma_{V,c}u^{N/(6N-8)}}{2\sigma_{Z,c}(u+(1-u)t)^{2(N-1)/(3N-4)}}, \qquad t \in (0,1),$$

and

$$\lambda_c = \frac{2}{(N+1)^2}\lambda_{1-} = \frac{\sqrt{N}\,\sigma_{V,c}u^{N/(6N-8)}}{(N+1)\sigma_{Z,c}}.$$

Each insider $i = 1, \dots, N$ follows

$$\begin{aligned}\theta_t^{i,*} &= \frac{(N+1)\sigma_{Z,m}^2}{2\sqrt{N}\,\sigma_{V,c}\sigma_{Z,c}u^{N/(6N-8)}(u+(1-u)t)^{(N-2)/(3N-4)}}(V-P_t), \qquad t \in (0,1),\\ \Delta X_1^{i,*} &= \frac{\sigma_{Z,c}}{\sqrt{N}\,\sigma_{V,c}u^{N/(6N-8)}}(V-P_{1-}).\end{aligned}$$

The residual variance is $\Sigma_0 = \sigma_{V,c}^2$,

$$\Sigma_t = \frac{\sigma_{V,c}^2 u^{N/(3N-4)}}{(u+(1-u)t)^{N/(3N-4)}}, \quad t \in (0,1), \qquad \Sigma_1 = \frac{\Sigma_{1-}}{N+1} = \frac{\sigma_{V,c}^2 u^{N/(3N-4)}}{N+1}. \tag{3.9}$$

Finally, we record the combination of Corollaries 3.4 and 3.5 corresponding to one insider and no opening auction.

Corollary 3.6. *Suppose that $N = 1$ in the setting of Corollary 3.5. Then $u = 1 + \frac{\sigma_{Z,m}^2}{\sigma_{Z,c}^2}$ and the equilibrium price impact coefficients become*

$$\lambda_t = \lambda_m := \frac{\sigma_{V,c}}{\sqrt{\sigma_{Z,m}^2+\sigma_{Z,c}^2}}, \quad t \in (0,1), \qquad \lambda_c = \frac{\lambda_m}{2} = \frac{\sigma_{V,c}}{2\sqrt{\sigma_{Z,m}^2+\sigma_{Z,c}^2}}.$$

In particular, price impact is constant during the continuous trading period.

The insider's trading strategy is

$$\begin{aligned}\theta_t^* &= \frac{\sigma_{Z,m}^2}{\lambda_m(\sigma_{Z,c}^2+\sigma_{Z,m}^2(1-t))}(V-P_t), \qquad t \in (0,1),\\ \Delta X_1^* &= \frac{V-P_{1-}}{2\lambda_c},\end{aligned}$$

and the residual variance is $\Sigma_0 = \sigma_{V,c}^2$,

$$\Sigma_t = \lambda_m^2\big(\sigma_{Z,c}^2+\sigma_{Z,m}^2(1-t)\big), \quad t \in (0,1), \qquad \Sigma_1 = \frac{\Sigma_{1-}}{2} = \frac{\lambda_m^2\sigma_{Z,c}^2}{2}.$$

4 Discussion

The derived equilibrium connects liquidity across the trading day to the timing of information and the availability of future trading opportunities. The opening and closing auctions play different economic roles in our model. The opening auction is the only opportunity for insiders to

exploit overnight information, which is publicized immediately after its conclusion, but they have future trading opportunities to profit from their insider knowledge of the persistent component. Conversely, the close is their final chance to trade on their private information. Continuous trading connects these two auctions, and the resulting equilibrium is rich enough to allow for high opening price impact, increasing liquidity over the day and a liquid closing auction, while leaving fundamental uncertainty unresolved at the end of the trading day.

The distinction between overnight and persistent information is central to understanding each insider's behaviour in the opening auction. Each insider's opening trade (3.5) decomposes into a linear combination of two terms, respectively exploiting the market maker's mispricing of the overnight and persistent components of the fundamental value. The ratio of these weights satisfies

$$\frac{\alpha_c}{\alpha_o} = 1 - \sigma_{Z,R} x^* \sqrt{1 - x^*} \in (0, 1).$$

This shows that the insiders trade on both components in the opening auction, but with a higher weight placed on the overnight component and explains why some of the persistent component's variance is reduced as a result of the opening auction.

Next, we investigate liquidity during and immediately after the opening auction. In practice, the opening auction is less liquid than the other trading periods (see Goyal et al. [2025]), so it is of interest to derive parameter regimes where the model obtains this ordering. The equilibrium values imply

$$\frac{\lambda_o}{\lambda_{0+}} = \frac{\sigma_{Z,R}}{2}\left(\sqrt{1 - x^*} + \sqrt{1 - x^* + \tfrac{4N\sigma^2_{V,o}}{\sigma^2_{V,c}(N+1)^2}}\right),$$

from which it is easy to see that

$$\lambda_o \geq \lambda_{0+} \iff \frac{4N\sigma^2_{V,o}}{\sigma^2_{V,c}(N+1)^2} \geq \frac{4}{\sigma^2_{Z,R}} - \frac{4\sqrt{1 - x^*}}{\sigma_{Z,R}}. \tag{4.1}$$

From here, we deduce that a simple sufficient condition for the opening auction to be less liquid than the start of the continuous trading period is if the condition

$$\sigma^2_{Z,R}\sigma^2_{V,R} \geq \frac{(N+1)^2}{N}$$

holds, where we set $\sigma_{V,R} = \sigma_{V,o}/\sigma_{V,c}$. This condition encapsulates two features: that information revealed after the opening auction is large relative to the information that persists during the rest of the trading day, and that noise trading volume in the opening auction is sufficiently small compared to the other periods. The product of the two ratios measuring these effects needs to be sufficiently large, so one effect, if sufficiently strong, can in principle explain the opening auction's illiquidity even if the other is not prominent. However, if $\sigma_{V,o} = 0$; i.e., no information is revealed after the opening auction, then (4.1) fails to hold and, as such, the opening auction is never less liquid than the start of the continuous trading period. This highlights that overnight news is a key driver of illiquidity in the opening auction.

The public announcement of V_o also changes how the opening order flow ΔY_0 should be interpreted. Once V_o is observed, the market maker readjusts their filtering from the opening auction order flow ΔY_0 alone, partially reducing this component's weight in favour of the newly revealed overnight information V_o. Indeed, from (2.2) we see that the coefficient in front of ΔY_0 changes from λ_o to $\lambda_o + \kappa_r$, which is a smaller positive weight since $-\lambda_o < \kappa_r < 0$, as can be deduced from

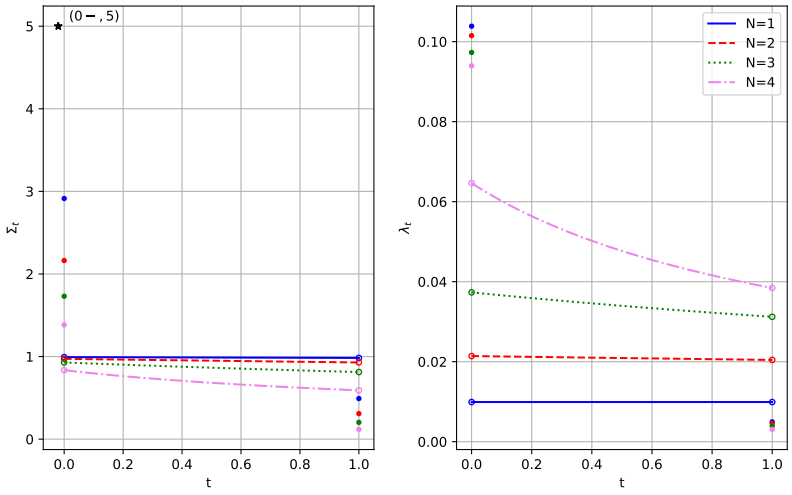

Figure 1: A schematic representation of the residual variance Σ (left panel) and price impact λ (right panel) in equilibrium for different numbers of insiders N present in the market with parameter values $\sigma_{V,o} = 2, \sigma_{V,c} = 1, \sigma_{Z,o} = \sigma_{Z,m} = 10, \sigma_{Z,c} = 100$. *Left panel:* the black star represents the common starting variance Σ_{0-}, while the filled points represent Σ_0 and Σ_1 with the continuous curves depicting $(\Sigma_t, t \in (0,1))$. *Right panel:* the filled points at zero and one represent λ_o and λ_c respectively, while the continuous curves plot $(\lambda_t, t \in (0,1))$.

the formulas in Theorem 3.2. The reason a positive weight on ΔY_0 remains after the overnight news is revealed is due to the fact that this order flow contains information about the persistent component V_c. The post-opening price revision thus reflects both the new public information and a reassessment of what earlier trades revealed.

At the end of the trading day, we see that the closing auction changes the incentives to reveal information during continuous trading. Its presence leaves a profitable final trading opportunity, and as a result the equilibrium trading-rate coefficient in the continuous trading period remains bounded as the close approaches. From Proposition 3.3, we see that the residual variance decreases during and after the opening auction, then continuously decreases throughout the trading day. At the closing auction a substantial fraction $N/(N+1)$ of the uncertainty remaining immediately before it is revealed, but the fundamental price V is not revealed completely. A schematic representation of this evolution is depicted in the left panel of Figure 1. This distinguishes the model from the continuous-trading benchmark in which prices converge to the fundamental value at the terminal time.

Liquidity in the continuous trading period and closing auction follows a similar profile. Once the opening auction is complete no further information is revealed, so the market maker improves their estimate over the remaining persistent component V_c exclusively through filtering. The closing auction then reveals a large fraction of the remaining uncertainty, leading to a sharp improvement in liquidity,

$$\lambda_c = \frac{2}{(N+1)^2}\lambda_{1-}.$$

For one insider, continuous-time impact is constant and the closing auction halves it. For multiple

insiders, impact declines during continuous trading and falls to the fraction $2/(N+1)^2$ of the impact level before the closing auction, as schematically depicted in the right panel of Figure 1. The auction therefore advances price discovery, while providing lower marginal price impact than the immediately preceding market.

Competition between insiders strengthens the fraction of residual uncertainty revealed at the close, but in the presence of too many insiders, all else held equal, the equilibrium itself may cease to exist. Indeed, for $N \geq 2$, the constructed equilibrium requires the parameter condition (3.1) to hold. The role of sufficiently large closing noise volume therefore makes it suboptimal for the insiders to front-run each other by fully revealing the fundamental value in the continuous trading period, which is a phenomenon that occurs when extending the standard continuous-time Kyle model of Back [1992] to more than one insider with full information. This model suggests that disclosure of overnight news can make the opening auction particularly costly, while the closing auction allows for low impact to coexist with incomplete information revelation. This provides a theoretical interpretation of the auction-versus-continuous-market empirical evidence of Goyal et al. [2025].

5 Calibration to equity data

We now calibrate the version of our model that omits the opening auction. Our dataset consists of daily volume and price data for 44 equities for the calendar year 2022. Specifically, we have the start-of-day price, the final observed price before the closing auction commences and the market-clearing price during the closing auction, as well as signed trading volume over the continuous trading period.

We use the results of Corollary 3.5 to derive consistent estimators for the parameters $\sigma_{V,c}$, $\sigma_{Z,m}$ and $\sigma_{Z,c}$. To this end, for each day and equity we consider the net signed continuous flow and price changes

$$Q = Y_{1-} - Y_0, \qquad C = P_{1-} - P_0, \qquad A = P_1 - P_{1-}.$$

Let q, c, and a denote the sample variances of these respective quantities across our one year of data. In equilibrium, aggregate continuous flow is a martingale in the market maker's filtration, with quadratic variation $\sigma^2_{Z,m}t$. Hence, $\mathrm{Var}(Q) = \sigma^2_{Z,m}$, even though observed flow includes insider trades. The variance of price changes can be expressed in terms of the residual variance Σ as follows:

$$\mathrm{Var}(C) = \sigma^2_{V,c} - \Sigma_{1-}, \qquad \mathrm{Var}(A) = \Sigma_{1-} - \Sigma_1 = \frac{N}{N+1}\Sigma_{1-}. \tag{5.1}$$

Matching the model moments to the sample variances gives

$$\sigma^2_{Z,m} = q, \qquad \sigma^2_{V,c} = c + \frac{N+1}{N}a, \qquad \Sigma_{1-} = \frac{N+1}{N}a. \tag{5.2}$$

From (3.9) we have that $\Sigma_{1-} = \sigma^2_{V,c}u^{N/(3N-4)}$. Recalling the definition of u in (3.2), solving for $\sigma_{Z,c}$ and using (5.2) leads to the estimators

$$\widehat{\sigma}^2_{Z,m} = q, \qquad \widehat{\sigma}^2_{V,c} = c + \frac{N+1}{N}a, \qquad \widehat{\sigma}^2_{Z,c} = \frac{q(N+1)^2(3N-4)(Nc+(N+1)a)^{(3N-4)/N}}{4((Nc+(N+1)a)^{(3N-4)/N} - ((N+1)a)^{(3N-4)/N})}. \tag{5.3}$$

Strikingly, the calibrated parameters always satisfy (3.1). Indeed, we have from (5.3) that

$$\frac{\widehat{\sigma}^2_{Z,c}}{\widehat{\sigma}^2_{Z,m}} = \frac{(N+1)^2(3N-4)}{4} \times \frac{(Nc+(N+1)a)^{(3N-4)/N}}{(Nc+(N+1)a)^{(3N-4)/N} - ((N+1)a)^{(3N-4)/N}} > \frac{(N+1)^2(3N-4)}{4}.$$

In practice, the number of insiders present in any given market is unknown. As such, Figure 2 depicts for $N \in \{1,2,3,4\}$, and for two randomly selected stocks in our dataset, the equilibrium values for $(\Sigma_t, t \in [0,1])$, $(\lambda_t, t \in [0,1))$ and λ_c according to Corollary 3.5 with parameters calibrated using (5.3). In all cases the larger the number of insiders the smaller the calibrated initial residual variance Σ_{0+}. For these stocks, the calibrated residual variance curves do not cross, with larger assumed numbers of insiders yielding lower residual uncertainty throughout the day. These comparisons concern separately calibrated models, rather than changes in competition holding the other parameters fixed, since the calibration is repeated for each value of N. In contrast, the price impact curves do cross over time. When more insiders are present in the market, the market maker sets a higher initial impact coefficient λ_{0+} anticipating the toxic order flow to come. However, as the strategically competing insiders reveal more information, the market maker is able to lower their impact. In all cases, immediately before the closing auction the impact ordering has fully reversed with the market having more insiders being more liquid than the one with fewer insiders.

A Deriving the equilibrium

In this section we derive the symmetric equilibrium of Theorem 3.2. We first examine the insider's problem in Subsection A.1, then the perfect competition condition of the market maker in Subsection A.2, and the equilibrium conditions in Subsection A.3. Finally, Subsection A.4 ties together the derivations of the other subsections and proves Theorem 3.2 as well as Corollaries 3.4-3.6.

A.1 Solving the insider's problem

Each insider's problem can be solved in closed form. We work backwards by first solving for the optimal trade $\Delta X_1^{i,*}$, measurable at time $1-$, to place in the closing auction. By analyzing the final term in the objective function (2.1), we see that

$$\begin{aligned}\mathbb{E}[(V-P_1)\Delta X_1^i \mid \mathcal{F}_1^I] &= \mathbb{E}\Big[\Big(V - P_{1-} - \lambda_c\big(\Delta X_1^i + \textstyle\sum_{j\neq i}\Delta X_1^j + \Delta Z_1\big)\Big)\Delta X_1^i \,\Big|\, \mathcal{F}_1^I\Big] \\ &= \mathbb{E}\Big[\Big(V - P_{1-} - \lambda_c\big(\Delta X_1^i + \textstyle\sum_{j\neq i}\Delta X_1^j\big)\Big)\Delta X_1^i \,\Big|\, \mathcal{F}_1^I\Big], \end{aligned} \tag{A.1}$$

where we used that ΔZ_1 is independent of $\mathcal{F}_1^I$ and has mean zero. This is quadratic in ΔX_1^i and all of the terms appearing in the expectation are $\mathcal{F}_1^I$-measurable. Hence, maximizing pointwise yields $\Delta X_1^{i,*} = \frac{1}{2\lambda_c}(V - P_{1-}) - \frac{1}{2}\sum_{j\neq i}\Delta X_1^j$. Imposing a symmetric equilibrium $\Delta X_1^{i,*} = \Delta X_1^{j,*}$ for all $i \neq j$ yields

$$\Delta X_1 := \Delta X_1^{i,*} = \frac{V - P_{1-}}{(N+1)\lambda_c}, \qquad i = 1,\dots,N. \tag{A.2}$$

Substituting back into (A.1) and recalling (2.1), we obtain the following representation for the value function when $t > 0$:

$$v(t,p,V_o,V) = \sup_{\theta^i\in\Theta_t} \mathbb{E}\Big[\int_t^1 (V-P_s)\theta_s^i ds + \frac{1}{(N+1)^2\lambda_c}(V-P_{1-})^2 \,\Big|\, \mathcal{F}_t^I\Big] = \sup_{\theta^i\in\Theta_t} J(t,p,V_o,V,\theta^i),$$

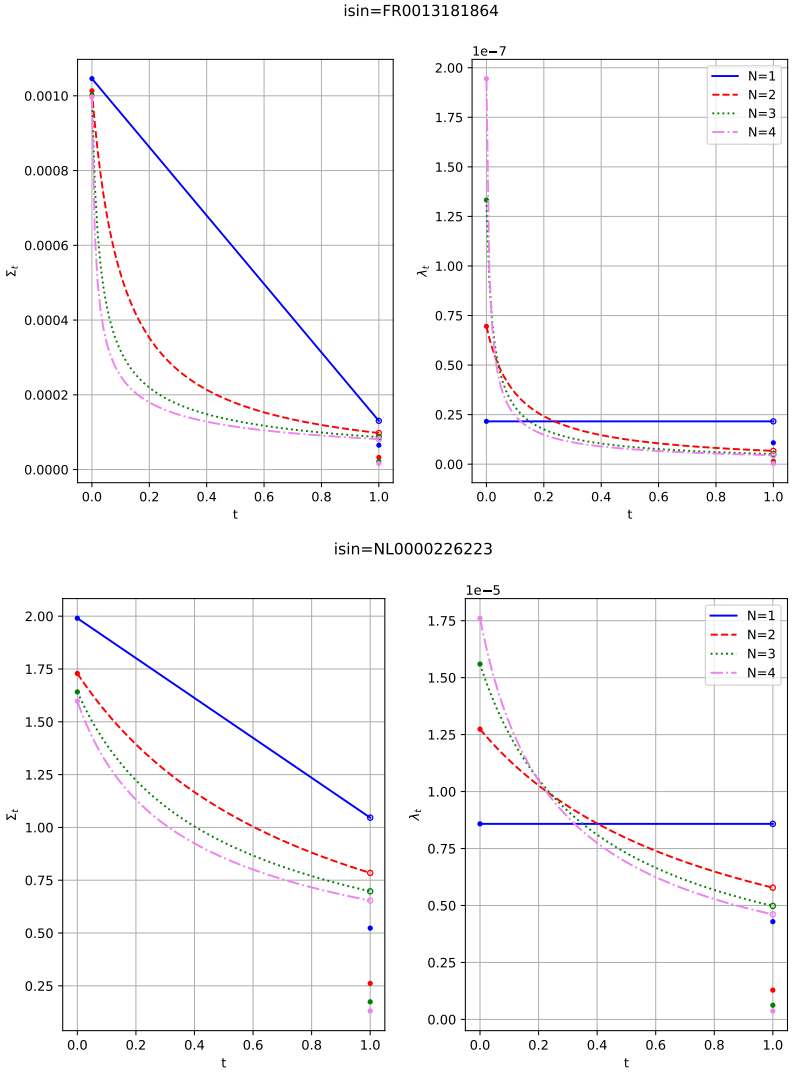

Figure 2: Residual variance and price impact evolution plots for two different randomly selected stocks (top and bottom) for different assumptions on the number of insiders N. Depicted are the model-predicted residual variance (left panel) and model-predicted price impact (right panel) with parameters calibrated according to (5.3). Continuous curves represent the quantities on $[0, 1)$, while the filled points at $t = 1$ represent Σ_1 (left) and λ_c (right).

where in the last equality we used the abuse of notation of replacing the last argument X^i of the insider's objective (2.1) by its trading rate in the continuous trading period θ^i. The set Θ_t consists of all square-integrable $\mathcal{F}^I$-adapted processes $(\theta^i_s;\, t \le s \le 1)$. Next, we assume λ is absolutely continuous on $(0,1)$ and that all the other insiders use the same linear strategy $\theta^j_t = \beta_t(V - P_t)$ for some deterministic function $(\beta_t; t \in (0,1))$. Then, we obtain

$$\begin{aligned}\frac{1}{2\lambda_{1-}}(P_{1-} - V)^2 &= \frac{1}{2\lambda_t}(P_t - V)^2 - \int_t^1 \frac{(P_s - V)^2}{2\lambda_s^2}\dot{\lambda}_s ds + \int_t^1 \frac{P_s - V}{\lambda_s} dP_s + \int_t^1 \frac{1}{2\lambda_s} d[P]_s \\ &= \frac{1}{2\lambda_t}(P_t - V)^2 - \int_t^1 \frac{(P_s - V)^2}{2\lambda_s^2}\dot{\lambda}_s ds + \int_t^1 (P_s - V)\theta^i_s ds \\ &\quad - (N-1)\int_t^1 \beta_s (P_s - V)^2 ds + \frac{\sigma^2_{Z,m}}{2}\int_t^1 \lambda_s ds + \sigma_{Z,m}\int_t^1 (P_s - V) dB_s.\end{aligned}$$

As such, substituting in for $\int_t^1 (V - P_s)\theta^i_s ds$ gives

$$\begin{aligned}&J(t, p, V_o, V, \theta^i) \\ &\quad = \frac{1}{2\lambda_t}(V - p)^2 + \frac{\sigma^2_{Z,m}}{2}\int_t^1 \lambda_s ds \\ &\qquad + \left(\frac{1}{(N+1)^2\lambda_c} - \frac{1}{2\lambda_{1-}}\right)\mathbb{E}[(V - P_{1-})^2 \mid \mathcal{F}^I_t] - \int_t^1 \left(\frac{\dot{\lambda}_s}{2\lambda_s^2} + (N-1)\beta_s\right)\mathbb{E}[(V - P_s)^2 \mid \mathcal{F}^I_t] ds.\end{aligned} \tag{A.3}$$

The two final terms in (A.3) account for the insider's incentive to shift trading across time and between the continuous market and the closing auction. Since unbounded trading rates θ^i are admissible, a positive contribution from maintaining a pricing error $V - P_s$, corresponding to positive coefficients in front of the conditional expectations, invites the insider to scale up that error, potentially generating unbounded profits. Conversely, a penalty creates an incentive to eliminate the error through increasingly rapid trading, revealing the fundamental price V prior to the closing auction. Thus, a finite equilibrium with regular trading throughout the continuous period and a nontrivial closing trade naturally calls for both terms in (A.3) to vanish. This leads to the following ODE for λ:

$$\dot{\lambda}_s = -2(N-1)\beta_s\lambda_s^2, \qquad \lambda_{1-} = \frac{1}{2}(N+1)^2\lambda_c, \tag{A.4}$$

which has solution

$$\lambda_t = \frac{(N+1)^2\lambda_c}{2(1 - (N-1)(N+1)^2\lambda_c\int_t^1 \beta_s ds)}, \qquad t \in (0,1). \tag{A.5}$$

Under this specification we see that

$$v(t, p, V_o, V) = \frac{1}{2\lambda_t}(V - p)^2 + \frac{\sigma^2_{Z,m}}{2}\int_t^1 \lambda_s ds, \qquad t \in (0,1),$$

and this value is achieved by insider i when using *any* admissible continuous-time trading rate $(\theta^i_s;\, t \le s \le 1) \in \Theta_t$ since the objective J is flat. In particular, the symmetric choice $\theta^i_t = \beta_t(V - P_t)$ is valid.

Finally, we solve for $v(0-,\mu,V_o,V)$, noting that at the pre-opening auction time $t=0-$ the value function genuinely depends on V_o. For each admissible opening order, the preceding calculation shows that any admissible trading rate, followed by the optimal closing order (A.2), achieves the continuation value $v(0+,P_{0+},V_o,V)$. Thus, by the tower property, the remaining optimization reduces to

$$v(0-,\mu,V_o,V)=\sup_{\Delta X_0^i\in\mathbb{R}}\mathbb{E}[(V-P_0)\Delta X_0^i+v(0+,P_{0+},V_o,V)\mid\mathcal{F}_{0-}^I],$$

which is also consistent with the dynamic programming principle. Recalling that

$$P_0=\mu+\lambda_o\big(\Delta X_0^i+\textstyle\sum_{j\neq i}\Delta X_0^j+\Delta Z_0\big),\quad P_{0+}=\mu+\kappa_o(V_o-\mu_o)+(\lambda_o+\kappa_r)\big(\Delta X_0^i+\textstyle\sum_{j\neq i}\Delta X_0^j+\Delta Z_0\big),$$

and that ΔZ_0 is independent of $\mathcal{F}_{0-}^I$, we have

$$\begin{aligned}
&v(0-,\mu,V_o,V)\\
&\quad=\sup_{\Delta X_0^i\in\mathbb{R}}\Bigg\{\mathbb{E}\big[\big(V-\mu-\lambda_o(\Delta X_0^i+\textstyle\sum_{j\neq i}\Delta X_0^j+\Delta Z_0)\big)\Delta X_0^i\,\big|\,\mathcal{F}_{0-}^I\big]+\frac{\sigma_{Z,m}^2}{2}\int_0^1\lambda_t dt\\
&\qquad\qquad+\mathbb{E}\bigg[\frac{1}{2\lambda_{0+}}\big(V-\mu-\kappa_o(V_o-\mu_o)-(\lambda_o+\kappa_r)(\Delta X_0^i+\textstyle\sum_{j\neq i}\Delta X_0^j+\Delta Z_0)\big)^2\,\big|\,\mathcal{F}_{0-}^I\bigg]\Bigg\}\\
&\quad=\sup_{\Delta X_0^i\in\mathbb{R}}\Bigg\{\bigg(\frac{(\lambda_o+\kappa_r)^2}{2\lambda_{0+}}-\lambda_o\bigg)(\Delta X_0^i)^2+\frac{\sigma_{Z,m}^2}{2}\int_0^1\lambda_t dt\\
&\quad+\bigg((V-\mu)\bigg(1-\frac{\lambda_o+\kappa_r}{\lambda_{0+}}\bigg)+\frac{\kappa_o(\lambda_o+\kappa_r)}{\lambda_{0+}}(V_o-\mu_o)+\bigg(\frac{(\lambda_o+\kappa_r)^2}{\lambda_{0+}}-\lambda_o\bigg)\textstyle\sum_{j\neq i}\Delta X_0^j\bigg)\Delta X_0^i\Bigg\}\\
&\qquad\qquad+\frac{1}{2\lambda_{0+}}\bigg(\big(V-\mu-\kappa_o(V_o-\mu_o)-(\lambda_o+\kappa_r)\textstyle\sum_{j\neq i}\Delta X_0^j\big)^2+(\lambda_o+\kappa_r)^2\sigma_{Z,o}^2\bigg).
\end{aligned}\tag{A.6}$$

We see that the objective is strictly concave and has a unique maximizer if

$$(\lambda_o+\kappa_r)^2<2\lambda_{0+}\lambda_o,\tag{A.7}$$

in which case it is given by

$$\begin{aligned}
\Delta X_0^{i,*}=&\frac{\lambda_{0+}-\lambda_o-\kappa_r}{2\lambda_o\lambda_{0+}-(\lambda_o+\kappa_r)^2}(V-\mu)+\frac{\kappa_o(\lambda_o+\kappa_r)}{2\lambda_o\lambda_{0+}-(\lambda_o+\kappa_r)^2}(V_o-\mu_o)\\
&+\frac{(\lambda_o+\kappa_r)^2-\lambda_o\lambda_{0+}}{2\lambda_o\lambda_{0+}-(\lambda_o+\kappa_r)^2}\textstyle\sum_{j\neq i}\Delta X_0^j.
\end{aligned}\tag{A.8}$$

Again imposing the symmetry condition $\Delta X_0^{j,*}=\Delta X_0^{i,*}$ for all $i\neq j$ gives

$$\Delta X_0:=\Delta X_0^{i,*}=\alpha_c(V_c-\mu_c)+\alpha_o(V_o-\mu_o),\qquad i=1,\dots,N,$$

where α_c, α_o are given by (3.7). The equilibrium of Theorem 3.2, which we construct in Section A.3, satisfies (A.7).

A.2 The market maker's problem

The market maker observes V_o at time 0+ as well as the total flow $Y_t = \sum_{i=1}^N X_t^i + Z_t$ for $t \in [0,1]$, and in a perfectly competitive fashion chooses the price $P_t = \mathbb{E}[V|\mathcal{F}_t^M]$ to be a martingale under their own filtration. As is usual, to find an equilibrium, we will assume for the continuous trading period that all of the insiders use the same linear strategy $\theta_t^i = \beta_t(V - P_t)$ for some deterministic function $(\beta_t;\, 0 < t < 1)$. In this case, the order flow for $t \in \mathbb{T} \setminus \{0-\}$ is

$$\begin{aligned} Y_t = N\alpha_o(V_o - \mu_o) + N\alpha_c(V_c - \mu_c) + \Delta Z_0 + \int_0^t N\beta_s(V - P_s)ds + \sigma_{Z,m}B_t \\ + \left(\frac{N(V - P_{1-})}{(N+1)\lambda_c} + \Delta Z_1\right)\mathbf{1}_{\{t=1\}}. \end{aligned} \tag{A.9}$$

Clearly $P_{0-} = \mu$, while the price during the opening auction satisfies

$$P_0 = \mathbb{E}[V \mid \mathcal{F}_0^M] = \mathbb{E}[V \mid \Delta Y_0].$$

Since ΔY_0 is given by the first three terms of (A.9), we see that $(V, \Delta Y_0)$ is a bivariate Gaussian random variable. Standard computations yield

$$P_0 = \mu + \rho_0 \Delta Y_0, \tag{A.10}$$

where

$$\rho_0 = \frac{\mathrm{Cov}(V, \Delta Y_0)}{\mathrm{Var}(\Delta Y_0)} = \frac{N\alpha_o\sigma_{V,o}^2 + N\alpha_c\sigma_{V,c}^2}{N^2\alpha_o^2\sigma_{V,o}^2 + N^2\alpha_c^2\sigma_{V,c}^2 + \sigma_{Z,o}^2}. \tag{A.11}$$

For $t \in (0,1)$, filtering leads to a continuous innovation process, as in the standard continuous-time Kyle model, but we must initialize it at P_{0+}. Using the martingale property we compute

$$P_{0+} = \mathbb{E}[V \mid \Delta Y_0, V_o] = V_o + \mu_c + \frac{N\alpha_c\sigma_{V,c}^2}{N^2\alpha_c^2\sigma_{V,c}^2 + \sigma_{Z,o}^2}\big(\Delta Y_0 - N\alpha_o(V_o - \mu_o)\big). \tag{A.12}$$

Indeed, this is most easily obtained from (A.9), which allows us to define the $\mathcal{F}_{0+}^M = \sigma(\Delta Y_0, V_o)$-measurable random variable $\widehat{Y}_0 := \Delta Y_0 - N\alpha_o(V_o - \mu_o) = N\alpha_c(V_c - \mu_c) + \Delta Z_0$. Since ΔZ_0 is independent of V_c, standard conditional formulas for $\mathbb{E}[V_c \mid \widehat{Y}_0]$ allow us to obtain (A.12), while standard conditional variance formulas for $\mathrm{Var}(V_c \mid \widehat{Y}_0)$ lead to

$$\Sigma_{0+} = \mathrm{Var}(V \mid \Delta Y_0, V_o) = \mathrm{Var}(V_c \mid \Delta Y_0, V_o) = \frac{\sigma_{V,c}^2\sigma_{Z,o}^2}{N^2\alpha_c^2\sigma_{V,c}^2 + \sigma_{Z,o}^2}. \tag{A.13}$$

With these initial conditions in place, we can apply standard Gaussian filtering to obtain the price $P_t = \mathbb{E}[V \mid \mathcal{F}_t^M]$ and its conditional variance $\Sigma_t = \mathrm{Var}(V \mid \mathcal{F}_t^M) = \mathrm{Var}(V_c \mid \mathcal{F}_t^M)$ for $t \in (0,1)$. Indeed, the Kalman–Bucy equations (see Kalman and Bucy [1961]) give

$$dP_t = \frac{N\Sigma_t\beta_t}{\sigma_{Z,m}^2}dY_t, \qquad \dot{\Sigma}_t = -\frac{N^2\beta_t^2}{\sigma_{Z,m}^2}\Sigma_t^2; \qquad\qquad t \in (0,1) \tag{A.14}$$

with initial conditions P_{0+} and Σ_{0+} given by (A.12) and (A.13) respectively.

We now determine P_1, which can be done with a direct hands-on approach. We have

$$P_1 = \mathbb{E}[V \mid \mathcal{F}_1^M] = \mathbb{E}[V \mid \mathcal{F}_{1-}^M, \Delta Y_1],$$

and since $V_c, V_o, \Delta Y_0, (Y_t;\, 0 \le t < 1)$ and ΔY_1 are jointly Gaussian random variables this can be computed explicitly. Working under the conditional law $\mathbb{P}(\cdot \mid \mathcal{F}_{1-}^M)$, we see that conditionally on $\mathcal{F}_{1-}^M$, the pair $(V, \Delta Y_1)$ is jointly Gaussian with

$$\mathbb{E}[V \mid \mathcal{F}_{1-}^M] = P_{1-}, \quad \mathbb{E}[\Delta Y_1 \mid \mathcal{F}_{1-}^M] = 0,$$

$$\mathrm{Var}(V \mid \mathcal{F}_{1-}^M) = \Sigma_{1-}, \quad \mathrm{Cov}(V, \Delta Y_1 \mid \mathcal{F}_{1-}^M) = \frac{N\Sigma_{1-}}{(N+1)\lambda_c}, \quad \mathrm{Var}(\Delta Y_1 \mid \mathcal{F}_{1-}^M) = \frac{N^2\Sigma_{1-}}{(N+1)^2\lambda_c^2} + \sigma_{Z,c}^2.$$

Hence, applying the standard conditioning formulas for bivariate Gaussian random variables yields

$$P_1 = \mathbb{E}[V \mid \mathcal{F}_{1-}^M, \Delta Y_1] = P_{1-} + \frac{N(N+1)\lambda_c\Sigma_{1-}}{N^2\Sigma_{1-} + (N+1)^2\sigma_{Z,c}^2\lambda_c^2}\Delta Y_1, \tag{A.15}$$

$$\Sigma_1 = \mathrm{Var}(V \mid \mathcal{F}_{1-}^M, \Delta Y_1) = \frac{(N+1)^2\lambda_c^2\sigma_{Z,c}^2\Sigma_{1-}}{(N+1)^2\lambda_c^2\sigma_{Z,c}^2 + N^2\Sigma_{1-}}. \tag{A.16}$$

A.3 Equilibrium

We now impose the equilibrium conditions. We start by connecting the closing auction to the continuous trading period.

A.3.1 Bridging the continuous trading period and closing auction

Starting with (A.15) we see that

$$\lambda_c = \frac{N(N+1)\lambda_c\Sigma_{1-}}{N^2\Sigma_{1-} + (N+1)^2\sigma_{Z,c}^2\lambda_c^2} \quad \Longrightarrow \quad \lambda_c = \frac{\sqrt{N\Sigma_{1-}}}{(N+1)\sigma_{Z,c}}. \tag{A.17}$$

Substituting into (A.16) then leads to $\Sigma_1 = \Sigma_{1-}/(N+1)$. Using the dynamics of P in (A.14) we obtain

$$\lambda_t = \frac{N\Sigma_t\beta_t}{\sigma_{Z,m}^2}, \qquad t \in (0,1). \tag{A.18}$$

Substituting this identity into the variance equation (A.14) and the ODE (A.4) for λ from the insider's problem gives

$$\dot{\Sigma}_t = -\sigma_{Z,m}^2\lambda_t^2, \qquad \dot{\lambda}_t = -\frac{2(N-1)}{N}\frac{\sigma_{Z,m}^2\lambda_t^3}{\Sigma_t}, \qquad t \in (0,1). \tag{A.19}$$

To solve this system, it is useful to introduce the quantity

$$H_t = \frac{\Sigma_t}{\lambda_t^2}, \qquad t \in (0,1).$$

Indeed, differentiating and applying (A.19) yields

$$\dot{H}_t = \frac{\dot{\Sigma}_t}{\lambda_t^2} - \frac{2\Sigma_t\dot{\lambda}_t}{\lambda_t^3} = -\sigma_{Z,m}^2 + \frac{4(N-1)}{N}\sigma_{Z,m}^2 = \frac{3N-4}{N}\sigma_{Z,m}^2.$$

Moreover, the terminal condition $\lambda_{1-} = \frac{1}{2}(N+1)^2\lambda_c$, together with (A.17), gives

$$H_{1-} = \frac{\Sigma_{1-}}{\lambda_{1-}^2} = \frac{4\sigma_{Z,c}^2}{N(N+1)^2}.$$

Integrating backwards from $1-$, we therefore obtain

$$H_t = \frac{4\sigma_{Z,c}^2}{N(N+1)^2} - \frac{3N-4}{N}\sigma_{Z,m}^2(1-t) = \frac{4\sigma_{Z,c}^2}{N(N+1)^2}\big(u+(1-u)t\big), \tag{A.20}$$

where u is as in (3.2). The requirement that H remain strictly positive, including at $0+$, is equivalent to $u > 0$. This corresponds precisely to the parameter condition (3.1).

Next, dividing the first equation in (A.19) by Σ_t and using (A.20), we obtain

$$\frac{\dot{\Sigma}_t}{\Sigma_t} = -\frac{\sigma_{Z,m}^2}{H_t} = -\frac{N}{3N-4}\frac{1-u}{u+(1-u)t}.$$

Integrating from $0+$ to t yields

$$\Sigma_t = \Sigma_{0+}\left(\frac{u}{u+(1-u)t}\right)^{N/(3N-4)}, \qquad t \in (0,1). \tag{A.21}$$

In particular, we have

$$\Sigma_{1-} = u^{N/(3N-4)}\Sigma_{0+}, \qquad \Sigma_1 = \frac{\Sigma_{1-}}{N+1}. \tag{A.22}$$

Substituting into (A.17) then gives

$$\lambda_c = \frac{\sqrt{N\Sigma_{0+}}}{(N+1)\sigma_{Z,c}}u^{N/(6N-8)}. \tag{A.23}$$

Finally, using $\lambda_t = \sqrt{\Sigma_t/H_t}$ and (A.18), we obtain

$$\beta_t = \frac{(N+1)\sigma_{Z,m}^2}{2\sqrt{N\Sigma_{0+}}\sigma_{Z,c}u^{N/(6N-8)}\big(u+(1-u)t\big)^{(N-2)/(3N-4)}}, \qquad t \in (0,1), \tag{A.24}$$

and

$$\lambda_t = \frac{(N+1)\sqrt{N\Sigma_{0+}}u^{N/(6N-8)}}{2\sigma_{Z,c}\big(u+(1-u)t\big)^{2(N-1)/(3N-4)}}, \qquad t \in (0,1). \tag{A.25}$$

A.3.2 Bringing in the opening auction

Taking $t \downarrow 0$ in (A.25) and rearranging for Σ_{0+} we obtain

$$\Sigma_{0+} = \frac{4\sigma_{Z,c}^2 u}{N(N+1)^2}\lambda_{0+}^2. \tag{A.26}$$

We now compare P_0 given by (A.10) with (2.2), and P_{0+} given by (A.12) with (2.2) to obtain

$$\lambda_o = \rho_0, \qquad \Lambda := \lambda_o + \kappa_r = \frac{N\alpha_c\sigma_{V,c}^2}{N^2\alpha_c^2\sigma_{V,c}^2 + \sigma_{Z,o}^2}, \qquad \kappa_o = 1 - \frac{N^2\alpha_o\alpha_c\sigma_{V,c}^2}{N^2\alpha_c^2\sigma_{V,c}^2 + \sigma_{Z,o}^2}. \tag{A.27}$$

Next, it is useful to introduce the quantity

$$x = \frac{\Sigma_{0+}}{\sigma_{V,c}^2} = \frac{\sigma_{Z,o}^2}{N^2\alpha_c^2\sigma_{V,c}^2 + \sigma_{Z,o}^2} \in (0,1), \tag{A.28}$$

which measures the proportion of the persistent component's variance remaining immediately after the conclusion of the opening auction. We will express all of the other equilibrium quantities $(\lambda_o, \Lambda, \lambda_{0+}, \lambda_c, \kappa_o, \alpha_o, \alpha_c)$ in terms of the model parameters and x.

From (A.26), the middle expression in (A.27) and (A.28) we can write[5]

$$\alpha_c = \frac{\sigma_{Z,o}}{N\sigma_{V,c}}\sqrt{\frac{1-x}{x}}, \qquad \Lambda = \frac{\sigma_{V,c}}{\sigma_{Z,o}}\sqrt{x(1-x)}, \qquad \lambda_{0+} = \frac{(N+1)\sqrt{N}\sigma_{V,c}}{2\sigma_{Z,c}\sqrt{u}}\sqrt{x}. \tag{A.29}$$

In particular, (A.29) gives

$$\frac{\Lambda}{\lambda_{0+}} = \sigma_{Z,R}\sqrt{1-x}, \qquad N\alpha_c\Lambda = 1-x, \tag{A.30}$$

where we recall $\sigma_{Z,R}$ of (3.2). Using the definition of α_c in (3.7), we obtain

$$\big((N+1)\lambda_o\lambda_{0+} - N\Lambda^2\big)\alpha_c = \lambda_{0+} - \Lambda.$$

Solving for λ_o and using the previously established identities yields

$$\lambda_o = \frac{\lambda_{0+} - \Lambda + N\alpha_c\Lambda^2}{(N+1)\alpha_c\lambda_{0+}} = \frac{1 - x\Lambda/\lambda_{0+}}{(N+1)\alpha_c} = \frac{N\sigma_{V,c}\sqrt{x}}{(N+1)\sigma_{Z,o}}\frac{1-\sigma_{Z,R}x\sqrt{1-x}}{\sqrt{1-x}}. \tag{A.31}$$

Moreover, the equations (3.7), (A.23) and (A.27) give

$$\lambda_c = \frac{\sqrt{N}\sigma_{V,c}\sqrt{x}}{(N+1)\sigma_{Z,c}}u^{N/(6N-8)}, \qquad \alpha_o = \frac{1}{(N+1)\lambda_o}, \qquad \kappa_o = 1 - N\alpha_o\Lambda.$$

From these identities we see that $\lambda_o, \lambda_c, \alpha_o$ and κ_o are all determined by x, as are the constants in (A.29).

Hence, to determine the equilibrium we just need to find the value of x. To this end, we use the remaining unused condition of (A.27), which imposes $\lambda_o = \rho_0$. Using (A.11), (A.28) and the identity $\alpha_o = 1/((N+1)\lambda_o)$ gives after rearranging

$$N\big((N+1)\lambda_o\alpha_c\big)^2 = (1-x)\bigg((N+1)^2\lambda_o\alpha_c + \frac{\sigma_{V,o}^2}{\sigma_{V,c}^2}\bigg).$$

Substituting $(N+1)\lambda_o\alpha_c = 1 - \sigma_{Z,R}x\sqrt{1-x}$ from (A.31) gives

$$N\big(1-\sigma_{Z,R}x\sqrt{1-x}\big)^2 = (1-x)\bigg((N+1)(1-\sigma_{Z,R}x\sqrt{1-x}) + \frac{\sigma_{V,o}^2}{\sigma_{V,c}^2}\bigg). \tag{A.32}$$

Taking the positive root of this quadratic equation in the variable $1-\sigma_{Z,R}x\sqrt{1-x}$, subtracting $1-x$ from both sides and creating a common denominator leads to the equation (3.3) for x. Note that the positive root is the relevant one because $1-\sigma_{Z,R}x\sqrt{1-x} = (N+1)\lambda_o\alpha_c > 0$. Although (3.3) does not give an explicit expression for x, it does have a unique solution. We record this result in the next proposition.

[5]Positive square roots were taken as the negative ones are incompatible with (A.7) and (A.27).

Proposition A.1. *The equation* (3.3) *has a unique solution* $x^* \in (0,1)$. *This solution additionally satisfies* $x^* > \max\{(1+\sigma_{V,R}^2)/(N+1+\sigma_{V,R}^2), 1-\sigma_{Z,R}^{-2}\}$, *where* $\sigma_{V,R} = \sigma_{V,o}/\sigma_{V,c}$.

Proof. Define

$$f(x) = x(1-\sigma_{Z,R}\sqrt{1-x}) + \frac{N-1}{2N}(1-x) - \frac{1}{2N}\sqrt{(N+1)^2(1-x)^2 + 4N\sigma_{V,R}^2(1-x)},$$

so that equation (3.3) becomes $f(x) = 0$. We have

$$f(0) = \frac{N-1}{2N} - \frac{1}{2N}\sqrt{(N+1)^2 + 4N\sigma_{V,R}^2} < 0, \qquad f(1) = 1 > 0,$$

so by continuity there exists a solution $x^* \in (0,1)$. To prove uniqueness, we compute that

$$f''(x) = \frac{\sigma_{Z,R}(4-3x)}{4(1-x)^{3/2}} + \frac{2N\sigma_{V,R}^4}{((N+1)^2(1-x)^2 + 4N\sigma_{V,R}^2(1-x))^{3/2}} > 0,$$

so that f is strictly convex. Since $f(0) < f(1)$ we see that either f is strictly increasing on $(0,1)$ or it is first strictly decreasing and then strictly increasing. In either case, since $f(0) < 0$, the function f can only have one root in $(0,1)$ proving uniqueness.

For the first bound on x^*, direct substitution gives

$$f\left(\frac{1+\sigma_{V,R}^2}{N+1+\sigma_{V,R}^2}\right) = -\sigma_{Z,R}\frac{1+\sigma_{V,R}^2}{N+1+\sigma_{V,R}^2}\sqrt{\frac{N}{N+1+\sigma_{V,R}^2}} < 0,$$

from which it follows that

$$x^* > \frac{1+\sigma_{V,R}^2}{N+1+\sigma_{V,R}^2}.$$

Next, note that since x^* satisfies $f(x^*) = 0$, we have that

$$2Nx^*(1-\sigma_{Z,R}\sqrt{1-x^*}) = \sqrt{(N+1)^2(1-x^*)^2 + 4N(1-x^*)\sigma_{V,o}^2/\sigma_{V,c}^2} - (N-1)(1-x^*) > 0,$$

from which we deduce that $x^* > 1-\sigma_{Z,R}^{-2}$. □

We now verify that the condition (A.7) holds, so that the equilibrium characterized by x^* is valid. From the first equality in (A.30) and the bound in Proposition A.1, we have

$$\frac{\Lambda}{\lambda_{0+}} = \sigma_{Z,R}\sqrt{1-x^*} < 1. \tag{A.33}$$

Additionally, from (A.32) we deduce the bound $N(1-\sigma_{Z,R}x^*\sqrt{1-x^*}) > (N+1)(1-x^*)$. From this estimate, (A.27) and (A.31) we obtain

$$\frac{\Lambda}{\lambda_o} = \frac{(N+1)(1-x^*)}{N(1-\sigma_{Z,R}x^*\sqrt{1-x^*})} < 1. \tag{A.34}$$

As such, the estimates (A.33) and (A.34) ensure $\Lambda^2/(\lambda_o\lambda_{0+}) < 1 < 2$, so that (A.7) holds.

A.4 Proof of Theorem 3.2 and Proposition 3.3

Using the derivations of the previous subsection we now rigorously establish Theorem 3.2.

Proof of Theorem 3.2 and Proposition 3.3. We need to verify that the conditions 1 and 2 of Definition 3.1 hold for the candidate equilibrium given in the statement of Theorem 3.2. First, we note that the price impact coefficients (3.4) in the continuous and closing auction periods satisfy (A.4) with

$$\beta_t = \frac{(N+1)\sigma_{Z,m}^2}{2\sqrt{N}\sigma_{V,c}\sigma_{Z,c}\sqrt{x^*}u^{N/(6N-8)}(u+(1-u)t)^{(N-2)/(3N-4)}}$$

being the deterministic pre-factor in front of the strategy θ in (3.6). As such, the insider's problem is within the scope of the calculations in Subsection A.1, which show that the symmetric strategy $(\Delta X_0, (\theta_t)_{t\in(0,1)}, \Delta X_1)$ stated in Theorem 3.2 is optimal for each insider to use, given that the other insiders use this same strategy. Since the equilibrium coefficients are bounded, the candidate strategy is clearly admissible. The computations of Subsections A.2 and A.3 now demonstrate that the price process P of (2.2) is an $\mathcal{F}^M$-martingale with the given equilibrium coefficients, establishing the theorem. The residual variance quantities in the statement of Proposition 3.3 follow from the identities (A.21), (A.22) and (A.26). □

We also note that Corollary 3.4 follows simply by setting $N = 1$ in the context of Theorem 3.2. Corollary 3.5 similarly follows by carrying out computations analogous to those in Subsections A.1-A.3, but starting at $t = 0+$ and omitting the opening auction. More directly, the equilibrium specified in Corollary 3.5 can be obtained by taking the limit $\sigma_{Z,o} \downarrow 0$ in the equilibrium quantities of Theorem 3.2. Finally, Corollary 3.6 is obtained by setting $N = 1$ in the equilibrium of Corollary 3.5.

References

Michael Aitken, Carole Comerton-Forde, and Alex Frino. Closing call auctions and liquidity. *Accounting & Finance*, 45(4):501–518, 2005. doi: 10.1111/j.1467-629X.2005.00155.x.

Kerry Back. Insider trading in continuous time. *The Review of Financial Studies*, 5(3):387–409, 1992.

Kerry Back, C. Henry Cao, and Gregory A. Willard. Imperfect competition among informed traders. *The Journal of Finance*, 55(5):2117–2155, 2000. doi: 10.1111/0022-1082.00282.

Stefanos Bazinas. NYSE closing auction: Price discovery opportunities reach new highs. NYSE Research, August 2024. URL `https://www.nyse.com/data-insights/nyse-closing-auction-price-discovery-opportunities-reach-new-highs`. Published August 29, 2024.

Paul Besson and Ludovic Quily. Better market impact at the close with residual imbalance. Euronext Quantitative Research, April 2026. URL `https://www.euronext.com/en/news/better-market-impact-close-residual-imbalance`. Published April 2, 2026.

Vincent Bogousslavsky and Dmitriy Muravyev. Who trades at the close? Implications for price discovery and liquidity. *Journal of Financial Markets*, 66:100852, 2023.

Carole Comerton-Forde and Barbara Rindi. Trading @ the close. Working Paper 3903757, SSRN, 2022. URL https://ssrn.com/abstract=3903757. Version dated September 28, 2022.

Rama Cont, Arseniy Kukanov, and Sasha Stoikov. The price impact of order book events. *Journal of financial econometrics*, 12(1):47–88, 2014.

F Douglas Foster and S Viswanathan. Strategic trading when agents forecast the forecasts of others. *The Journal of Finance*, 51(4):1437–1478, 1996.

Amit Goyal, Narasimhan Jegadeesh, and Yanbin Wu. Price impact in closing auctions, opening auctions, and continuous markets: A benchmark for cost of trading on anomalies. *Journal of Financial and Quantitative Analysis*, pages 1–36, 2025.

Craig W Holden and Avanidhar Subrahmanyam. Long-lived private information and imperfect competition. *The Journal of Finance*, 47(1):247–270, 1992.

Narasimhan Jegadeesh and Yanbin Wu. Closing auctions: Nasdaq versus NYSE. *Journal of Financial Economics*, 143(3):1120–1139, 2022. URL https://www.sciencedirect.com/science/article/pii/S0304405X21005092.

R. E. Kalman and R. S. Bucy. New results in linear filtering and prediction theory. *Journal of Basic Engineering*, 83(1):95–108, 1961. doi: 10.1115/1.3658902.

Eugene Kandel, Barbara Rindi, and Luisella Bosetti. The effect of a closing call auction on market quality and trading strategies. *Journal of Financial Intermediation*, 21(1):23–49, 2012. doi: 10.1016/j.jfi.2011.03.002.

Albert S. Kyle. Continuous auctions and insider trading. *Econometrica*, 53(6):1315–1335, 1985. doi: 10.2307/1913210.

Mohammed Salek, Damien Challet, and Ioane Muni Toke. Price impact in equity auctions: Zero, then linear, 2023. URL https://arxiv.org/abs/2301.05677.

Mohammed Salek, Damien Challet, and Ioane Muni Toke. Equity auction dynamics: Latent liquidity models with activity acceleration. *Quantitative Finance*, 24(10):1381–1398, 2024. doi: 10.1080/14697688.2024.2367680.

Avanidhar Subrahmanyam. Transaction taxes and financial market equilibrium. *The Journal of Business*, 71(1):81–118, 1998.